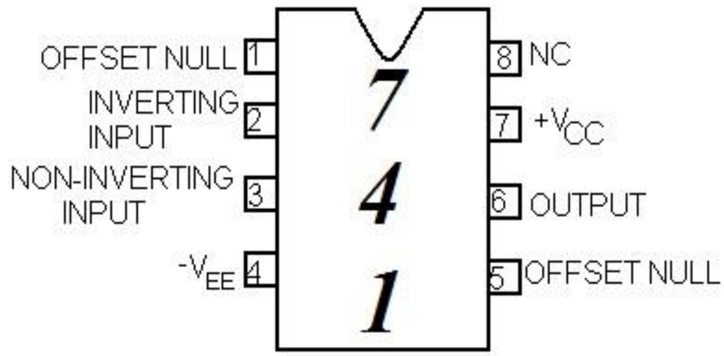
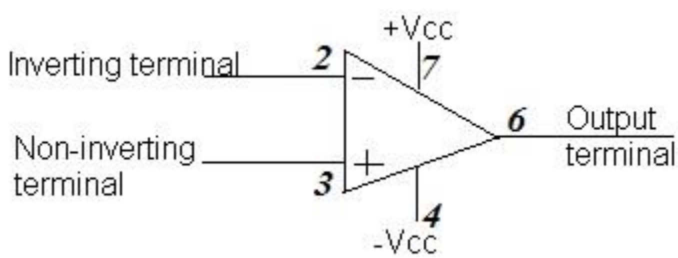
OPERATIONAL AMPLIFIER (OP-AMP)

- The operational amplifier is a very high gain, multistage, negative feedback differential amplifier with high input impedance & low output impedance.
- Earlier Op-Amps were constructed with vacuum tubes and were primarily used to perform mathematical operations such as addition, subtraction, differentiation & integration – thus the term operational. They required high voltage for operation.
- Today's Op-Amps are linear integrated circuits that use relatively low voltage.
- It is an 8-PIN Integrated Circuit(IC) having 2 - input terminals ,1- output terminal ,2-terminals for D.C supply, 2-terminals for Off Set and one terminal for ground.

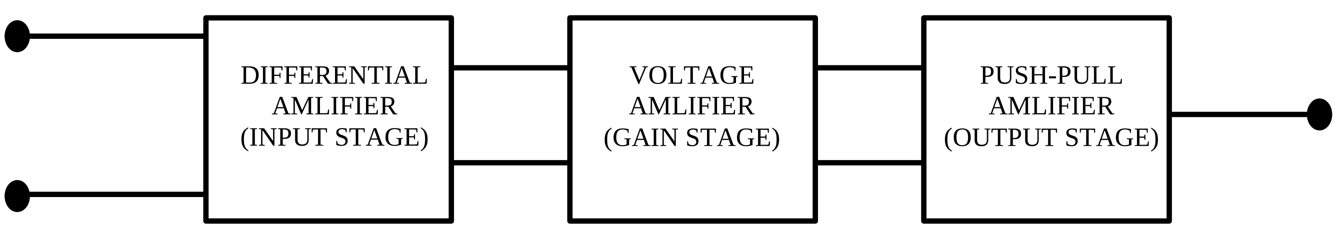
Pin diagram



Symbol of OP-AMP

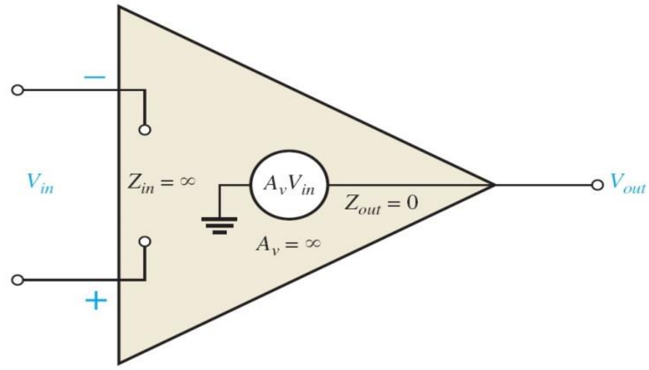


Internal Block Diagram of an OP-AMP



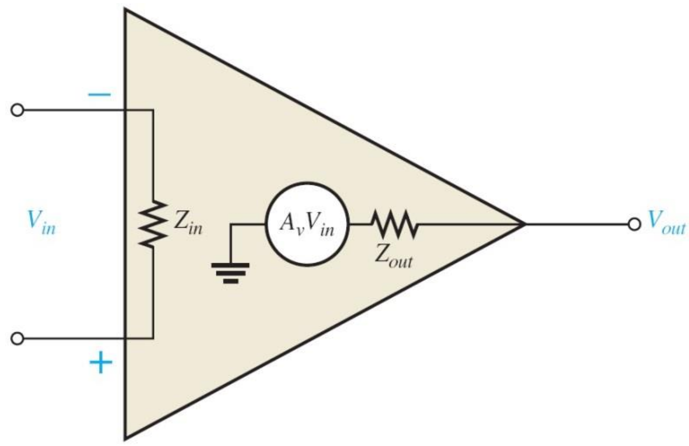
- A typical op-amp is made up of three types of amplifier circuits: a differential amplifier, a voltage amplifier, and a push-pull amplifier, as shown in the figure.
- The differential amplifier is the input stage for the op-amp. It provides amplification of the difference voltage between the two inputs.
- The second stage is usually a class-A amplifier that provides additional gain. Some op-amps may have more than one voltage amplifier stage.
- A push-pull class B amplifier is typically used for the output stage. It increases the output voltage swing and raises the current supplying capabilities of op-amp.

Characteristics of Ideal OP-AMP



- The open loop gain of ideal op-amp is infinite.
- The input impedance is infinite.
- The output impedance is zero.
- The output voltage is zero for zero input voltage.
- Common Mode Rejection Ratio (CMRR) is infinite.
- Slew Rate (SR) is infinite.
- Infinite frequency bandwidth.

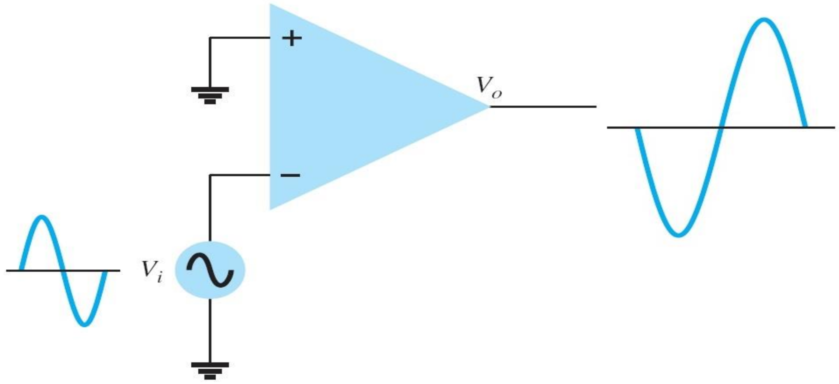
Characteristics of Practical OP-AMP



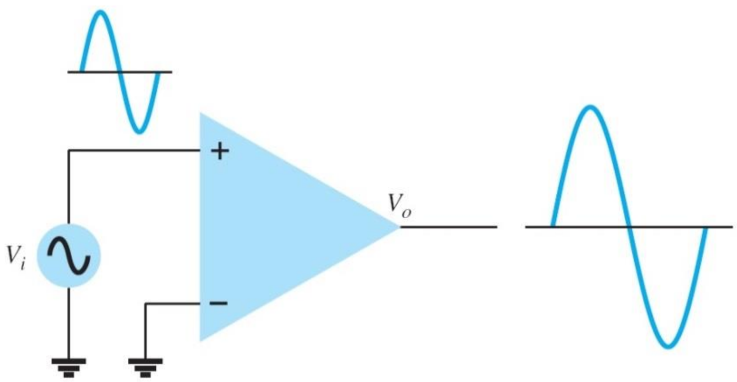
- The open loop gain of ideal op-amp is very high.
- The input impedance is very high.
- The output impedance is very low.
- The output voltage may or may not be zero for zero input voltage.
- Common Mode Rejection Ratio (CMRR) is very high.
- Slew Rate (SR) is very high.

Open Loop Configuration

1. **Single Ended Input operation:**
   When input signal is applied to any of the input terminal of the op-amp with the other input terminal at ground potential, it is called as single ended input operation.

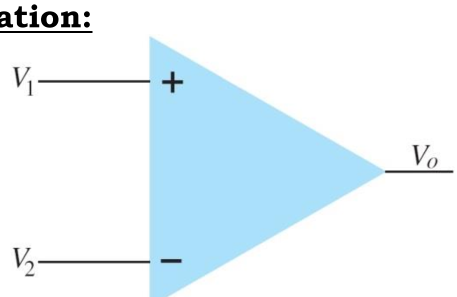


When input is applied to negative input terminal with positive input terminal is being grounded, the configuration is called as inverting configuration. Here, the output is 180^0 out of phase to that of the applied input.



When input is applied to positive input terminal with negative input terminal is being grounded, the configuration is called as non-inverting configuration. Here, the output is in phase with the applied input.

2. Double Ended Input operation:



Here the input is applied to both the terminals simultaneously and the output of the configuration is given by

$$V_O = A_d V_d + A_c V_c$$

Where

A_d = Differential Gain

A_c = Common mode Gain

V_d = Difference Voltage = $|V_1 - V_2|$

V_c = Common mode Gain = $\frac{V_1 + V_2}{2}$

Common Mode Rejection Ratio(CMRR)

CMRR is defined as the ratio of Differential Gain over the Common Mode Gain.

Mathematically, $CMRR = \frac{A_d}{A_c}$

We know that,

$$V_o = A_d V_d + AcVc$$

$$\Rightarrow V_o = A_d V_d \left[1 + \frac{AcVc}{A_d V_d}\right]$$

$$\Rightarrow V_o = A_d V_d \left[1 + \frac{1}{CMRR}\frac{Vc}{V_d}\right]$$

The CMRR can be expressed in terms of Decibel (dB) as

$$CMRR(dB) = 20\log_{10}\left(\frac{A_d}{A_c}\right)$$

Examples:

1. Calculate the differential gain of an Opamp having common mode gain=100 And CMRR=60 dB

Soln: - $CMRR = 60dB = 20\log_{10}(A_d/A_c)$ dB

⇨ $60dB = 20\log_{10}(A_d/A_c)$ dB

⇨ $60 = 20\log_{10}(A_d/A_c)$

⇨ $3 = \log_{10}(A_d/A_c)$

⇨ $A_d/A_c = 10^3 = 1000$

Again, $A_c = 1000$; $A_d = 1000 \times A_c = 1,00,000$

- Therefore the differential gain of the Opamp is 1,00,000

2. Calculate the Output voltage in dB of an Opamp having differential gain 10,000 and CMRR=1,000and also having the two input signals V1= 2mV and V2 = 5mV

Soln: - The total output voltage in terms of CMRR can be represented as

$V_O = AdVd\,[1+V_C / (CMRR \times V_D)]$

And Ad =10,000, CMRR=1,000

$V_D = V2 - V1 = 5mV - 2mV = 3mV$

$V_C = (V1+V2) / 2 = (2mV+5mV) / 2 = 3.5mV$

⇨ $V_o = A_d V_d[1+V_c/(CMRR \times V_d)]$

$= 10,000 \times 3mV\,[1+3.5mV/(1,000 \times 3mV)]$

$= 30[1+ (3.5 / (3 \times 1000)]$

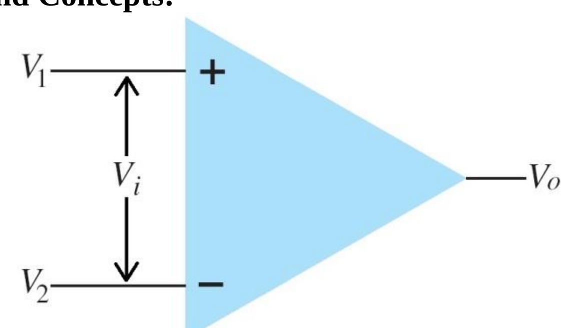
$= 30(0.00117) = 30.035$

V_O (in dB) $= 20\log_{10}(30.035)$ dB $= 29.55$dB

Closed Loop Configuration

NOTE:

Virtual Short & Virtual Ground Concepts:



For any amplifier, $V_0 = A.V_i$

Where, V_0 = Output of the amplifier

V_i = Input of the amplifier

A = Gain of the amplifier

For Op-Amp, $V_i = V_1 - V_2$

The gain of the ideal op-amp is infinite. Let us assume that the op-amp is working properly and produces a finite output. Hence the voltage between the op-amp input terminals should be negligibly small and nearly equal to zero i.e.

$$\mathrm{V_i} \approx 0$$

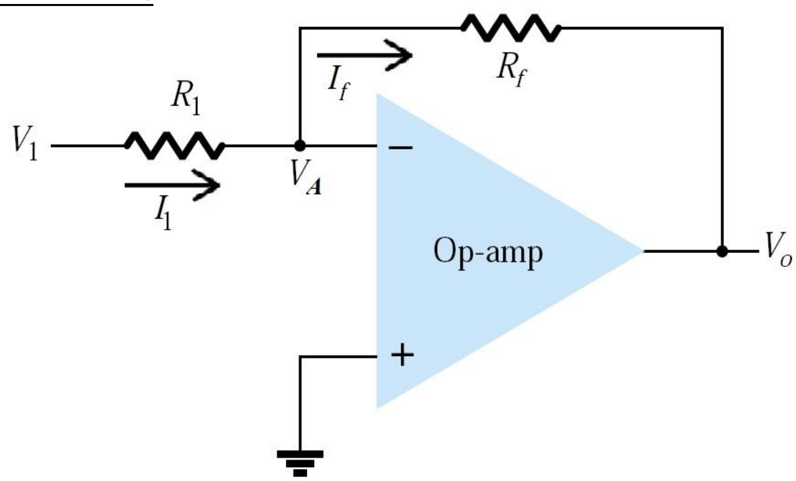
$$\Rightarrow V_1 - V_2 \approx 0$$

$$\Rightarrow V_1 \approx V_2$$

Hence we can say there exist a short circuit in between the two input terminals. This is a virtual short and the two terminals are not shorted physically. This concept is called as virtual short concept.

When one of the terminals is at ground potential, then this potential can also be observed at the other terminal. This is called as virtual ground concept.

1. INVERTING AMPLIFIER:



The most commonly used constant-gain amplifier circuit is the inverting amplifier, as shown in the figure. Here the input is applied to the negative input terminal & positive input terminal is connected to ground. The output is obtained by multiplying the input by a fixed or constant gain, set by the input resistor (R_1) and feedback resistor (R_f). This output is also inverted from the input.

By KCL,

$$I_1 = I_f$$

$$\Rightarrow \frac{V_1 - V_A}{R_1} = \frac{V_A - V_0}{R_f}$$

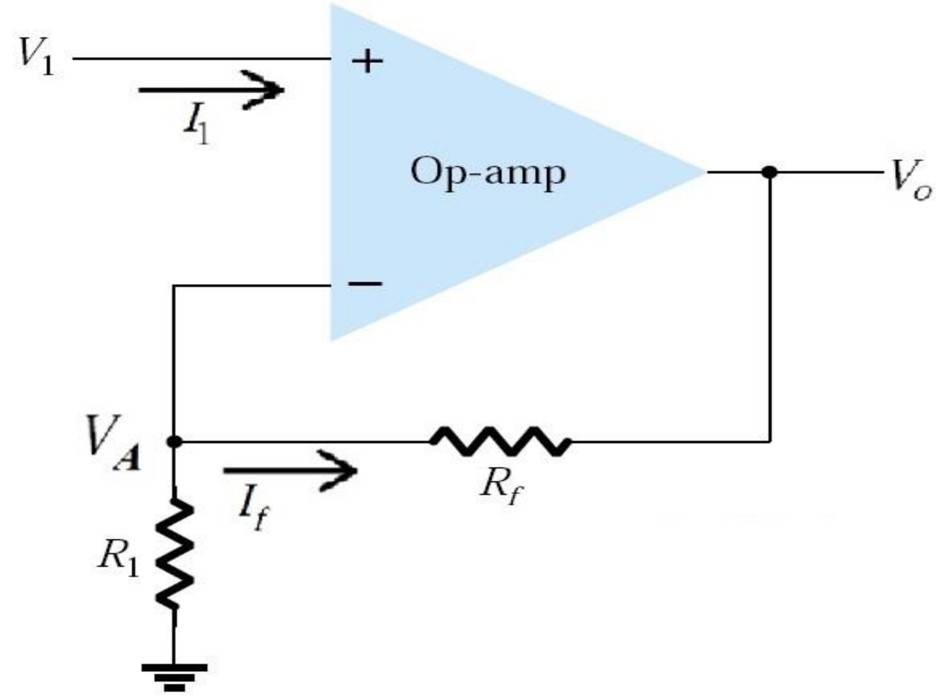
Due to virtual ground concept $V_A = 0$.

$$\Rightarrow \frac{V_1}{R_1} = -\frac{V_0}{R_f}$$

$$\Rightarrow A = \frac{V_0}{V_1} = -\frac{R_f}{R_1}$$

$$\Rightarrow V_0 = -\frac{R_f}{R_1} V_1$$

2. NON-INVERTING AMPLIFIER:



Here the input is applied to the positive input terminal & negative input terminal is connected to ground. This op-amp circuit works as a non-inverting amplifier or constant-gain multiplier. It should be noted that the inverting amplifier connection is more widely used because it has better frequency stability.

By KCL,

$$I_1 = I_f$$

$$\Rightarrow -\frac{V_A}{R_1} = \frac{V_A - V_0}{R_f}$$

Due to virtual short concept $V_A = V_1$.

$$\Rightarrow -\frac{V_1}{R_1} = \frac{V_1 - V_0}{R_f}$$

$$\Rightarrow -\frac{V_1}{R_1} = \frac{V_1}{R_f} - \frac{V_0}{R_f}$$

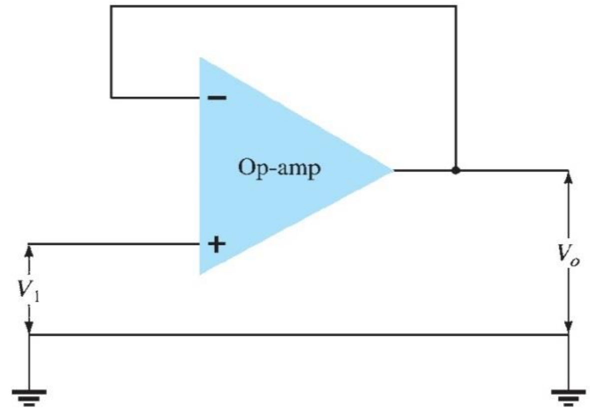
$$\Rightarrow \frac{V_0}{R_f} = \left[\frac{V_1}{R_f} + \frac{V_1}{R_1}\right]$$

$$\Rightarrow V_0 = \left[\frac{V_1}{R_f} + \frac{V_1}{R_1}\right] R_f$$

$$\Rightarrow V_0 = V_1\left[1 + \frac{R_f}{R_1}\right]$$

$$\Rightarrow A = \frac{V_0}{V_1} = 1 + \frac{R_f}{R_1}$$

3. UNITY FOLLOWER:

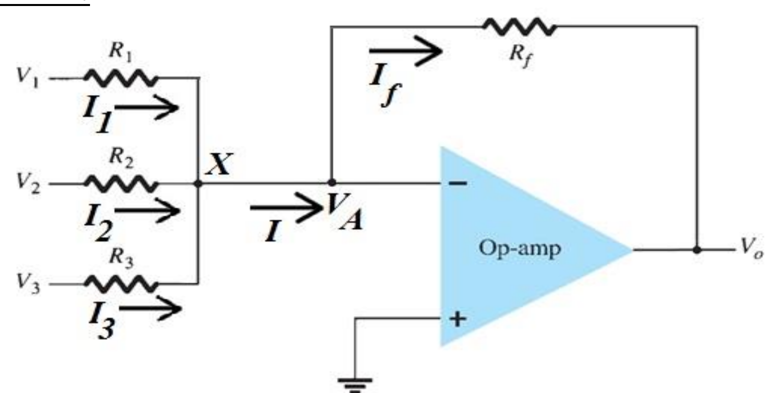


The unity-follower circuit provides a gain of unity (1) with no polarity or phase reversal.
Mathematically,

$$V_o = V_1$$

Here the output is of same polarity and magnitude as of the input.

4. SUMMING AMPLIFIER:



Summing amplifier is the most used op-amp circuit and is shown in the figure. The circuit shows a three-input summing amplifier, which provides a means of algebraically summing (adding) three voltages, each multiplied by a constant-gain factor. Each input adds a voltage to the output multiplied by its separate constant- gain multiplier.

Now, applying KCL at point X, we have

$$I = I_1 + I_2 + I_3$$

$$\Rightarrow I = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

Again by KCL,

$$I = I_f$$

$$\Rightarrow \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = \frac{V_A - V_0}{R_f}$$

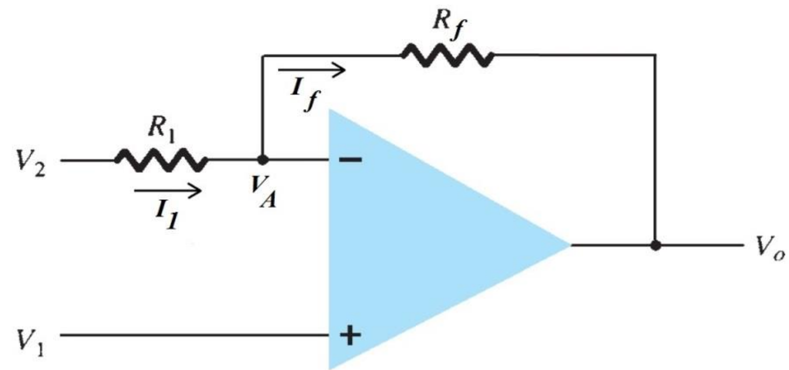
Due to virtual ground concept $V_A = 0$.

$$\Rightarrow \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_0}{R_f}$$

$$\Rightarrow V_0 = -R_f\left[\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right]$$

$$\Rightarrow V_0 = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right]$$

5. SUBTRACTOR:



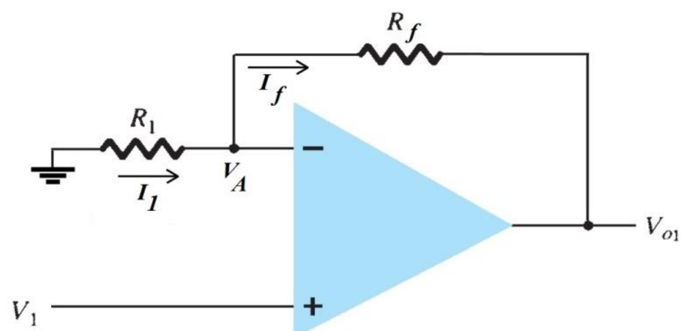
Here, by superposition theorem the output can be written as

$$V_0 = V_{01} + V_{02}$$

where, V_{01} = Output due to the input V_1 when V_2 is at ground potential
V_{02} = Output due to the input V_2 when V_1 is at ground potential

Calculation of V_{01}:

Considering the input V_1 and making the other input V_2 grounded the subtractor circuit can be drawn as,

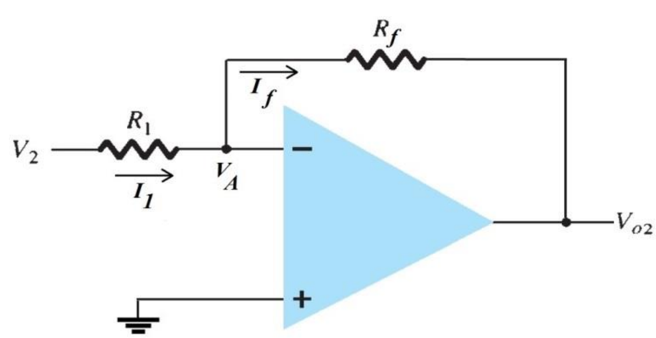


Now, the above circuit is a non-inverting amplifier circuit and its output is expressed as,

$$V_{01} = \left(1 + \frac{R_f}{R_1}\right)V_1$$

Calculation of V_{02}:

Considering the input V_2 and making the other input V_1 grounded the subtractor circuit can be drawn as,



Now, the above circuit is a non-inverting amplifier circuit and its output is expressed as,

$$V_{02} = -\frac{R_f}{R_1}V_2$$

Hence, the output of the subtractor circuit is given by

$$V_0 = V_{01} + V_{02}$$

$$\Rightarrow V_0 = \left(1 + \frac{R_f}{R_1}\right)V_1 - \frac{R_f}{R_1}V_2$$

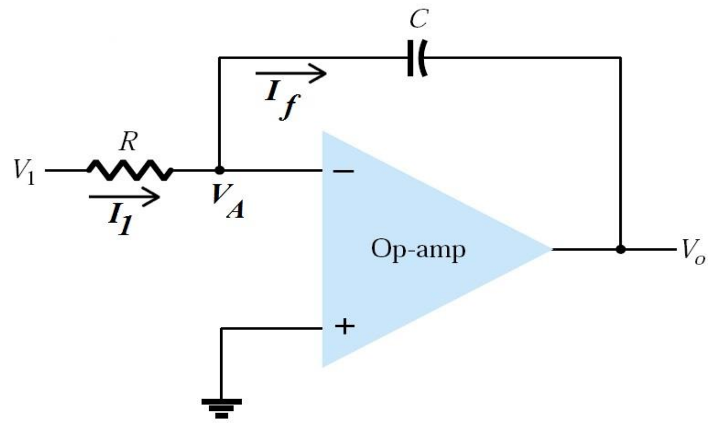
For a practical op-amp circuit $\frac{R_f}{R_1} >> 1$. Hence neglecting 1 with respect to $\frac{R_f}{R_1}$ we have,

$$V_0 \approx \frac{R_f}{R_1}V_1 - \frac{R_f}{R_1}V_2$$

$$\Rightarrow V_0 \approx \frac{R_f}{R_1}(V_1 - V_2)$$

6. INTEGRATOR:

Till now all the circuits have resistors as input and feedback components. If a capacitor is used as feedback component as shown in the figure, the resulting circuit is called as integrator.



We know that, by KCL

$$I_1 = I_f$$

$$\Rightarrow \frac{V_1 - V_A}{R} = C\frac{d(V_A - V_0)}{dt} \quad \left[\because \text{The current through a capacitor is expressed as } I = C\frac{dV}{dt}\right]$$

Due to virtual ground concept, $V_A = 0$.

$$\Rightarrow \frac{V_1}{R} = -C\frac{dV_0}{dt}$$

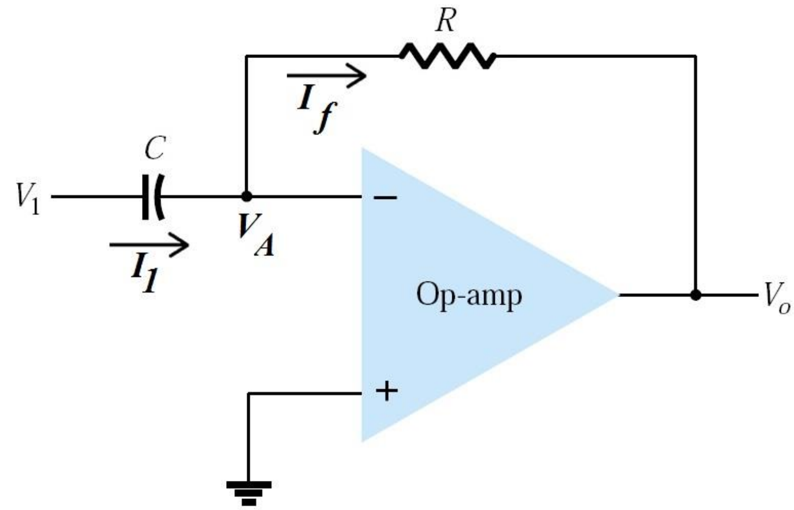
$$\Rightarrow \frac{dV_0}{dt} = -\frac{1}{RC}V_1$$

$$\Rightarrow dV_0 = -\frac{1}{RC}V_1 dt$$

$$\Rightarrow V_0 = -\frac{1}{RC}\int V_1 dt$$

7. DIFFERENTIATOR:

The differentiator circuit is shown below.



We know that, by KCL

$$I_1 = I_f$$

$$\Rightarrow C\frac{d(V_1 - V_A)}{dt} = \frac{V_A - V_0}{R}$$

Due to virtual ground concept, $V_A = 0$.

$$\Rightarrow C\frac{dV_1}{dt} = -\frac{V_0}{R}$$

$$\Rightarrow V_0 = -RC\frac{dV_1}{dt}$$

SLEW RATE (SR)

Slew Rate is the parameter reflecting the op-amp's ability to handle varying signals and is defined as maximum rate at which amplifier output can change in volts per microsecond (V/ms).

Mathematically,

$$SR = \left.\frac{dV_0}{dt}\right|_{\max} \qquad V/\mu s$$

The slew rate provides a parameter specifying the maximum rate of change of the output voltage when driven by a large step-input signal.

If one tried to drive the output at a rate of voltage change greater than the slew rate, the output would not be able to change fast enough and would not vary over the full range expected, resulting in signal clipping or distortion.

Example:

Q. For an OPAMP having the slew rate of SR=2V/μs, what is the maximum closed loop voltage gain that can be used when the input signal varies by 0.5V in 10μs.

Solution:

Slew Rate,SR=2V/μs

$$\frac{\Delta Vin}{\Delta t} = \frac{0.5V}{10\mu s} = 0.05V/\mu s$$

Since the output voltage $Vout = A_{CL} Vin$

$$\frac{\Delta Vout}{\Delta t} = A_{CL}\frac{\Delta Vin}{\Delta t}$$

$\therefore$ Closed –Loop gain, $A_{CL} = \frac{\Delta Vout/\Delta t}{\Delta Vin/\Delta t} = \frac{SR}{\Delta Vin/\Delta t}$

$$= \frac{2V/\mu s}{0.05V/\mu s} = 40$$

Examples:

1. If an integrator circuit of has R_1 = 100 kΩ and R_f =500 kΩ, what output voltage results for an input of V_1 = 2 V?

Ans: Given,

R_1 = 100 kΩ

R_f =500 kΩ

Inverting Circuit

For inverting circuit,

$$V_0 = -\frac{R_f}{R_1}V_1$$

$$\Rightarrow V_0 = -\frac{500k\Omega}{100k\Omega}\times 2V$$

$$\Rightarrow V_0 = -5\times 2V$$

$$\Rightarrow V_0 = -10V$$

2. Calculate the output voltage of a non-inverting amplifier for values of V_1 = 2 V, R_f = 500 kΩ and R_1 = 100 kΩ.

Ans: Given,

R_1 = 100 kΩ

R_f =500 kΩ

Non-inverting Circuit

For non-inverting circuit,

$$V_0 = \left[1+\frac{R_f}{R_1}\right]V_1$$

$$\Rightarrow V_0 = \left[1+\frac{500k\Omega}{100k\Omega}\right]\times 2V$$

$$\Rightarrow V_0 = 6\times 2V$$

$$\Rightarrow V_0 = 12V$$

3. Calculate the output voltage of an op-amp summing amplifier for the following sets of voltages and resistors. Use R_f = 1 MΩ in all cases.
   a. V_1 = +1 V, V_2 = +2 V, V_3 = +3 V, R_1 = 500 kΩ, R_2 = 1 MΩ, R_3 = 1 MΩ
   b. V_1 = -2 V, V_2 = +3 V, V_3 = +1 V, R_1 = 200 kΩ, R_2 = 500 kΩ, R_3 = 1 MΩ

Ans:

a. Given,

V_1 = +1 V, V_2 = +2 V, V_3 = +3 V
R_1 = 500 kΩ, R_2 = 1 MΩ, R_3 = 1 MΩ
R_f = 1MΩ

The output of summing amplifier is given by

$$V_0 = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right]$$
$$\Rightarrow V_0 = -\left[\frac{1M\Omega}{500k\Omega}\times 1V + \frac{1M\Omega}{1M\Omega}\times 2V + \frac{1M\Omega}{1M\Omega}\times 3V\right]$$
$$\Rightarrow V_0 = -7V$$

b. Given,

V_1 = -2 V, V_2 = +3 V, V_3 = +1 V
R_1 = 200 kΩ, R_2 = 500 kΩ, R_3 = 1 MΩ
R_f = 1MΩ

The output of summing amplifier is given by

$$V_0 = -\left[\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right]$$
$$\Rightarrow V_0 = -\left[\frac{1M\Omega}{200k\Omega}\times(-2V) + \frac{1M\Omega}{500k\Omega}\times 3V + \frac{1M\Omega}{1M\Omega}\times 1V\right]$$
$$\Rightarrow V_0 = 3V$$

4. Calculate the Output voltage of an inverting Amplifier which having gain is 80dB and Input signal is 20μV(p-p)

Ans: Given,

inverting amplifier
Gain = 80dB
V_1=20 μV

Now,

Gain = 80dB

$$\Rightarrow 20\log_{10}(|A|) = 80$$
$$\Rightarrow \log_{10}(|A|) = 4$$
$$\Rightarrow |A| = 10^4 = 10000$$
$$\Rightarrow -\frac{R_f}{R_1} = 10000$$
$$\Rightarrow -\frac{V_0}{20\mu V} = 10000$$
$$\Rightarrow V_0 = -10000\times 20\mu V = -0.2V$$

5. Calculate the Output voltage of a non-inverting Amplifier which having gain is 100dB and Input signal is 5μV(p-p).

Ans: Given,

Non-inverting amplifier
Gain = 100dB
$V_1 = 5\ \mu V$

Now,

Gain = 100dB

$$\Rightarrow 20\log_{10}(|A|) = 100$$

$$\Rightarrow \log_{10}(|A|) = 5$$

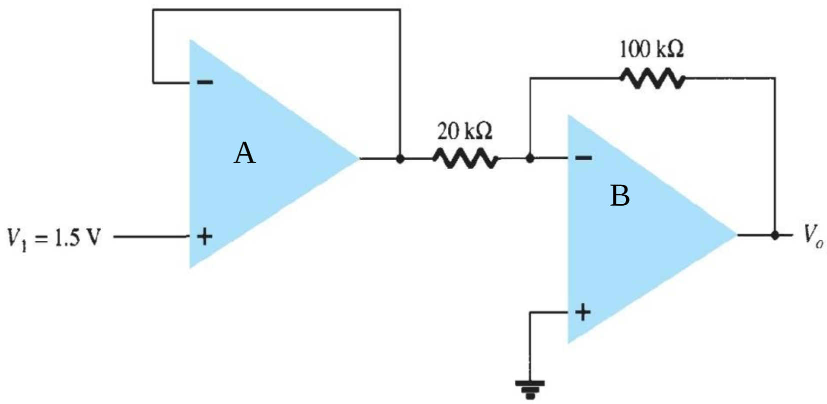
$$\Rightarrow |A| = 10^5$$

$$\Rightarrow 1 + \frac{R_f}{R_1} = 10^5$$

$$\Rightarrow \frac{V_0}{5\mu V} = 10^5$$

$$\Rightarrow V_0 = 10^5 \times 5\mu V = 0.5V$$

6. Calculate the output voltage of the given circuit.



Ans:

Op-amp – A is an unit follower circuit.
So its output is given by

$$V_0 = V_1 = 1.5V$$

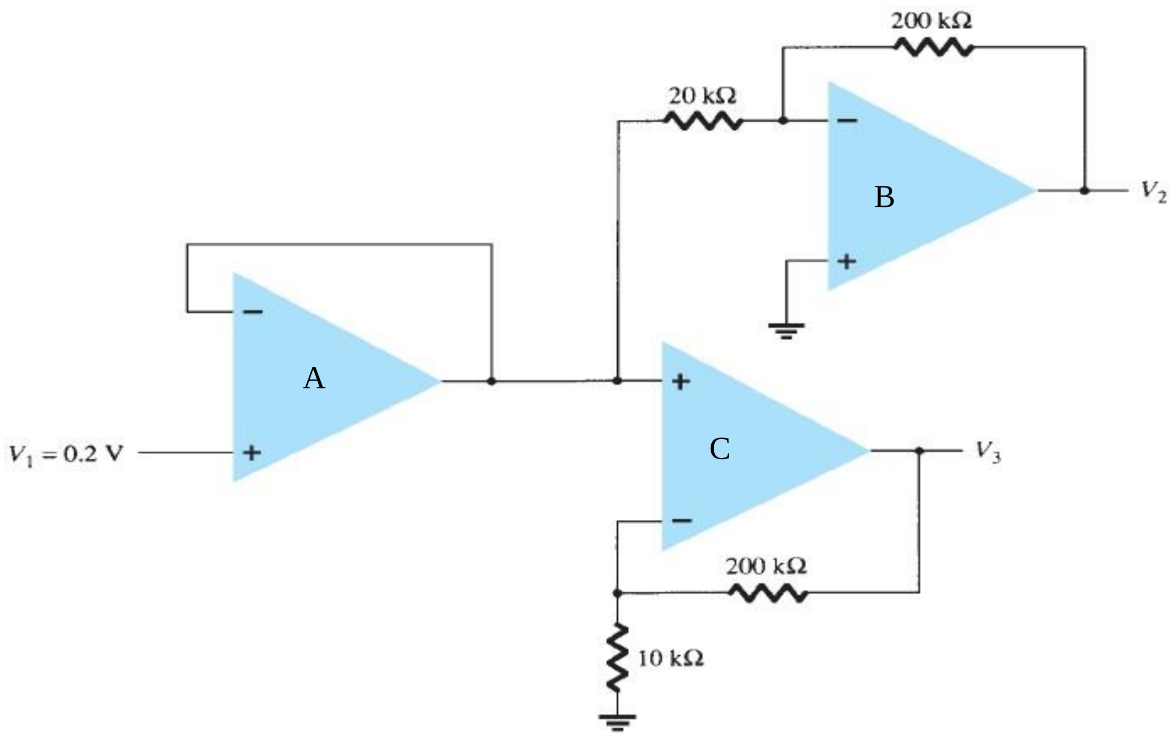
Now, Op-amp –B is an inverting amplifier circuit.

It's output is given by

$$V_0 = -\frac{R_f}{R_1}V_1$$

$$\Rightarrow V_0 = -\frac{100k\Omega}{20k\Omega} \times 1.5V = -7.5V$$

7. Calculate the output voltages of the given circuit.



Ans:

Op-amp – A: Unit Follower

Output is given by

$$V_0 = V_1 = 0.2V$$

Op-amp – B: Inverting Amplifier

It's output is given by

$$V_2 = -\frac{R_f}{R_1}V_1$$

$$\Rightarrow V_2 = -\frac{200k\Omega}{20k\Omega} \times 0.2V = -2V$$

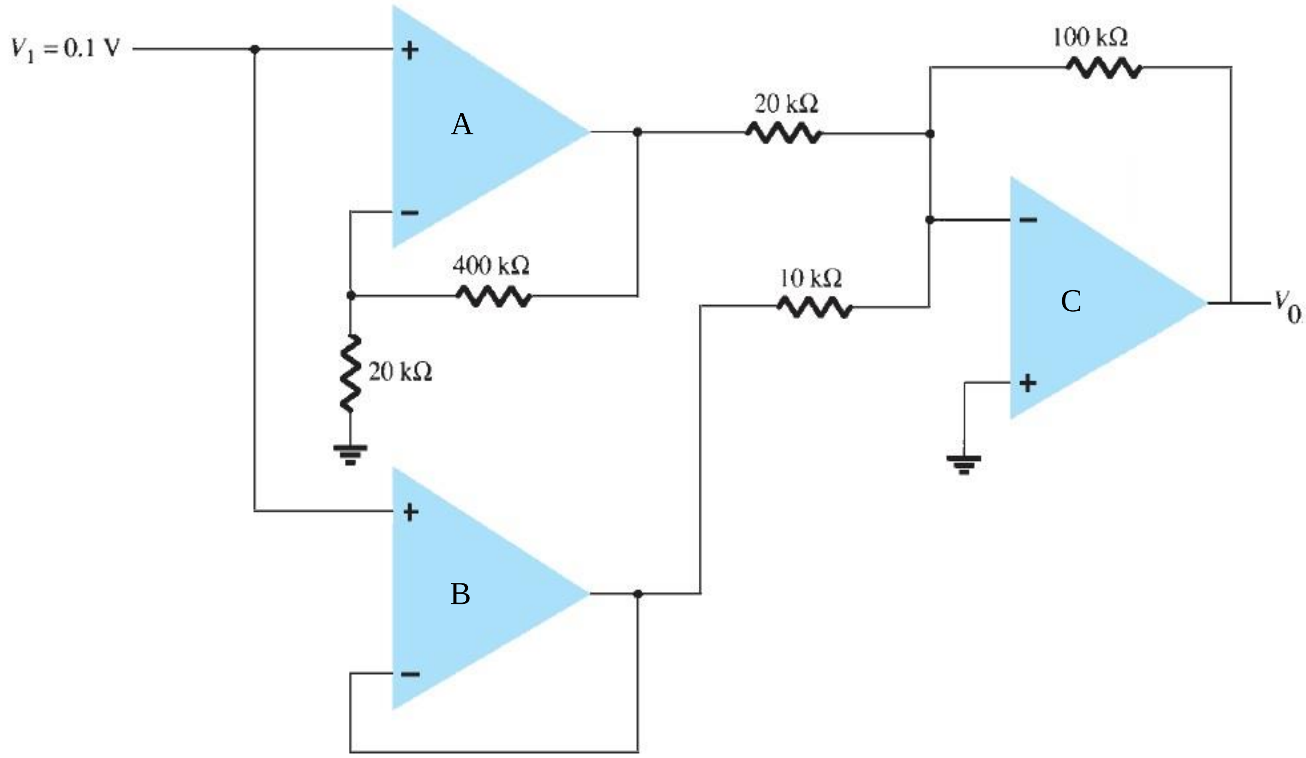
Op-amp – C: Non-inverting Amplifier

It's output is given by

$$V_3 = \left[1 + \frac{R_f}{R_1}\right]V_1$$

$$\Rightarrow V_3 = \left[1 + \frac{200k\Omega}{10k\Omega}\right] \times 0.2V = 4.2V$$

8. Calculate the output voltage of the given circuit.



Ans:

Op-amp – A: Non-inverting Circuit

Output is given by

$$V_0 = \left[1 + \frac{R_f}{R_1}\right] V_1$$

$$\Rightarrow V_0 = \left[1 + \frac{400k\Omega}{20k\Omega}\right] \times 0.1V$$

$$\Rightarrow V_0 = 2.1V$$

Op-amp – B: Unit Follower

Output is given by

$$V_0 = V_1 = 0.1$$

Op-amp – C: Summing Amplifier

It's output is given by

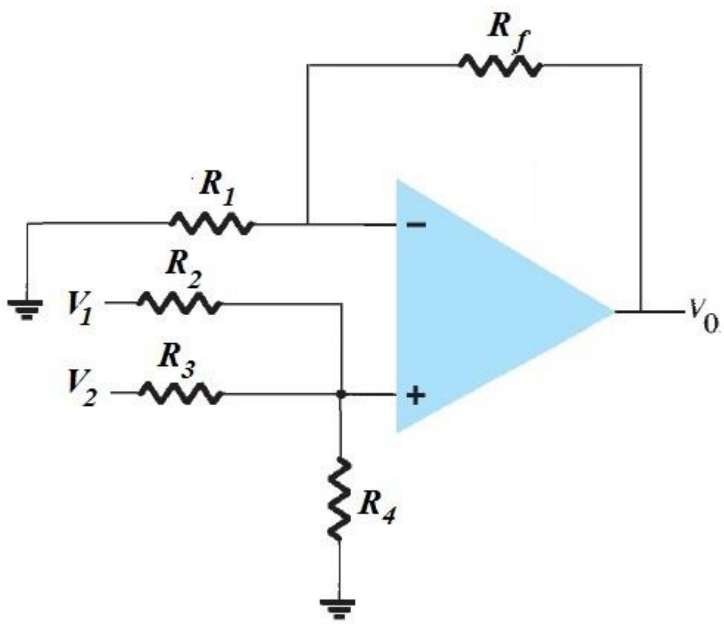
$$V_0 = -\left[\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2\right]$$

$$\Rightarrow V_0 = -\left[\frac{100k\Omega}{20k\Omega} \times 2.1V + \frac{100k\Omega}{10k\Omega} \times 0.1V\right]$$

$$\Rightarrow V_0 = -[10.5V + 1V]$$

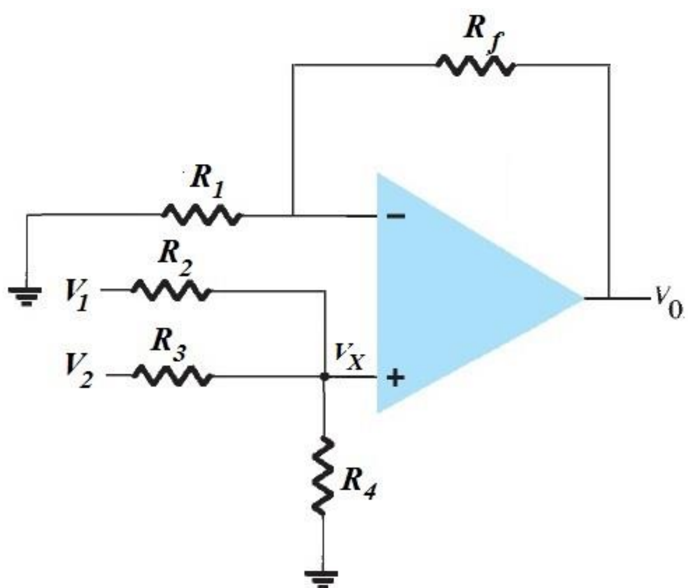
$$\Rightarrow V_0 = -11.5V$$

9. Calculate an expression for the output voltage of the given circuit.

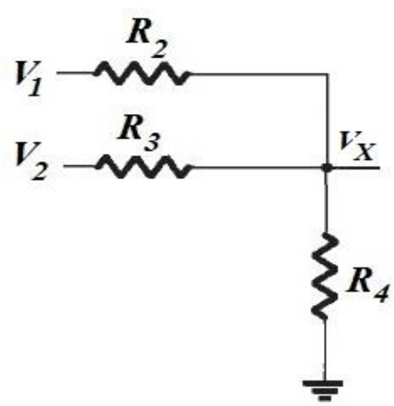


Ans:

The given circuit is



Let us first calculate the voltage V_X of the circuit. The circuit for calculating the V_X voltage is given by



Using super position theorem, the V_X voltage can be expressed as

$$V_X = V_{X1} + V_{X2}$$

Where, V_{X1} = Output due to V_1 when V_2 is at ground potential
V_{X2} = Output due to V_2 when V_1 is at ground potential

Now,

$$V_{X1} = \frac{R_3 \parallel R_4}{R_2 + (R_3 \parallel R_4)} V_1$$

&

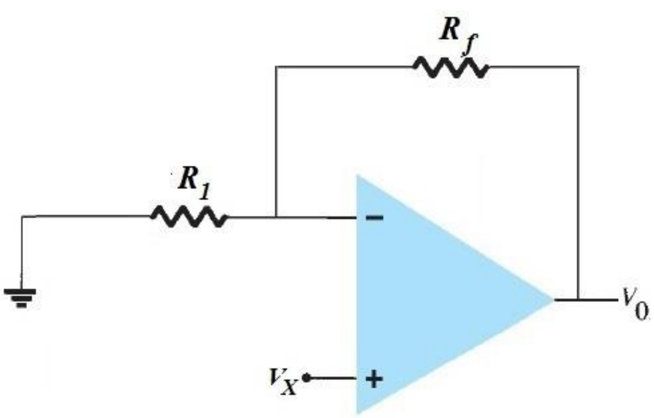
$$V_{X2} = \frac{R_2 \parallel R_4}{R_3 + (R_2 \parallel R_4)} V_2$$

Hence,

$$V_X = V_{X1} + V_{X2}$$

$$\Rightarrow V_X = \frac{R_3 \parallel R_4}{R_2 + (R_3 \parallel R_4)} V_1 + \frac{R_2 \parallel R_4}{R_3 + (R_2 \parallel R_4)} V_2$$

Now, the given circuit can be redrawn as,

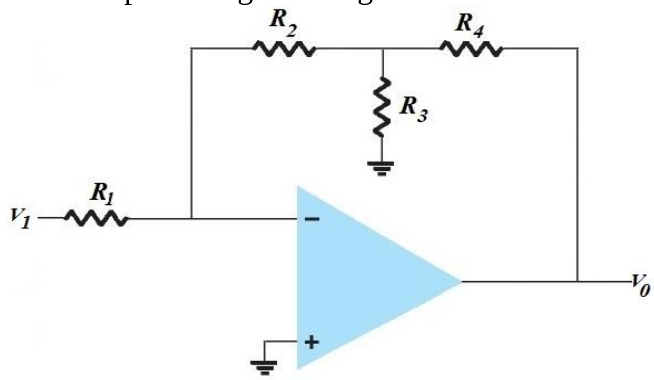


The above circuit is a non-inverting amplifier & its output is given by,

$$V_0 = \left(1 + \frac{R_f}{R_1}\right) V_X$$

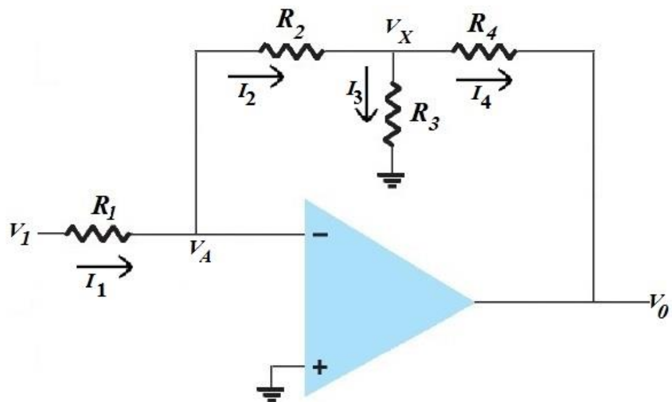
$$\Rightarrow V_0 = \left(1 + \frac{R_f}{R_1}\right)\left(\frac{R_3 \parallel R_4}{R_2 + (R_3 \parallel R_4)} V_1 + \frac{R_2 \parallel R_4}{R_3 + (R_2 \parallel R_4)} V_2\right)$$

10. Calculate an expression for the output voltage of the given circuit.



Ans:

The given circuit is



By KCL,

$$I_1 = I_2$$

$$\Rightarrow \frac{V_1 - V_A}{R_1} = \frac{V_A - V_X}{R_2}$$

Due to virtual ground, $V_A = 0$.

$$\Rightarrow \frac{V_1}{R_1} = -\frac{V_X}{R_2}$$

$$\Rightarrow V_X = -\frac{R_2}{R_1} V_1$$

Again by KCL,

$$I_2 = I_3 + I_4$$

$$\Rightarrow \frac{V_A - V_X}{R_2} = \frac{V_X}{R_3} + \frac{V_X - V_0}{R_4}$$

$$\Rightarrow -\frac{V_X}{R_2} = \frac{V_X}{R_3} + \frac{V_X}{R_4} - \frac{V_0}{R_4}$$

$$\Rightarrow \frac{V_0}{R_4} = \frac{V_X}{R_2} + \frac{V_X}{R_3} + \frac{V_X}{R_4}$$

$$\Rightarrow V_0 = \left[\frac{V_X}{R_2} + \frac{V_X}{R_3} + \frac{V_X}{R_4}\right] R_4$$

$$\Rightarrow V_0 = V_X \left[\frac{R_4}{R_2} + \frac{R_4}{R_3} + \frac{R_4}{R_4}\right]$$

$$\Rightarrow V_0 = V_X \left[1 + \frac{R_4}{R_3} + \frac{R_4}{R_2}\right]$$

Now, putting the value of V_X, we have

$$V_0 = \left(-\frac{R_2}{R_1}V_1\right)\left[1+\frac{R_4}{R_3}+\frac{R_4}{R_2}\right]$$

$$\Rightarrow V_0 = \left(-\frac{R_2}{R_1}\right)\left[1+\frac{R_4}{R_3}+\frac{R_4}{R_2}\right]V_1$$

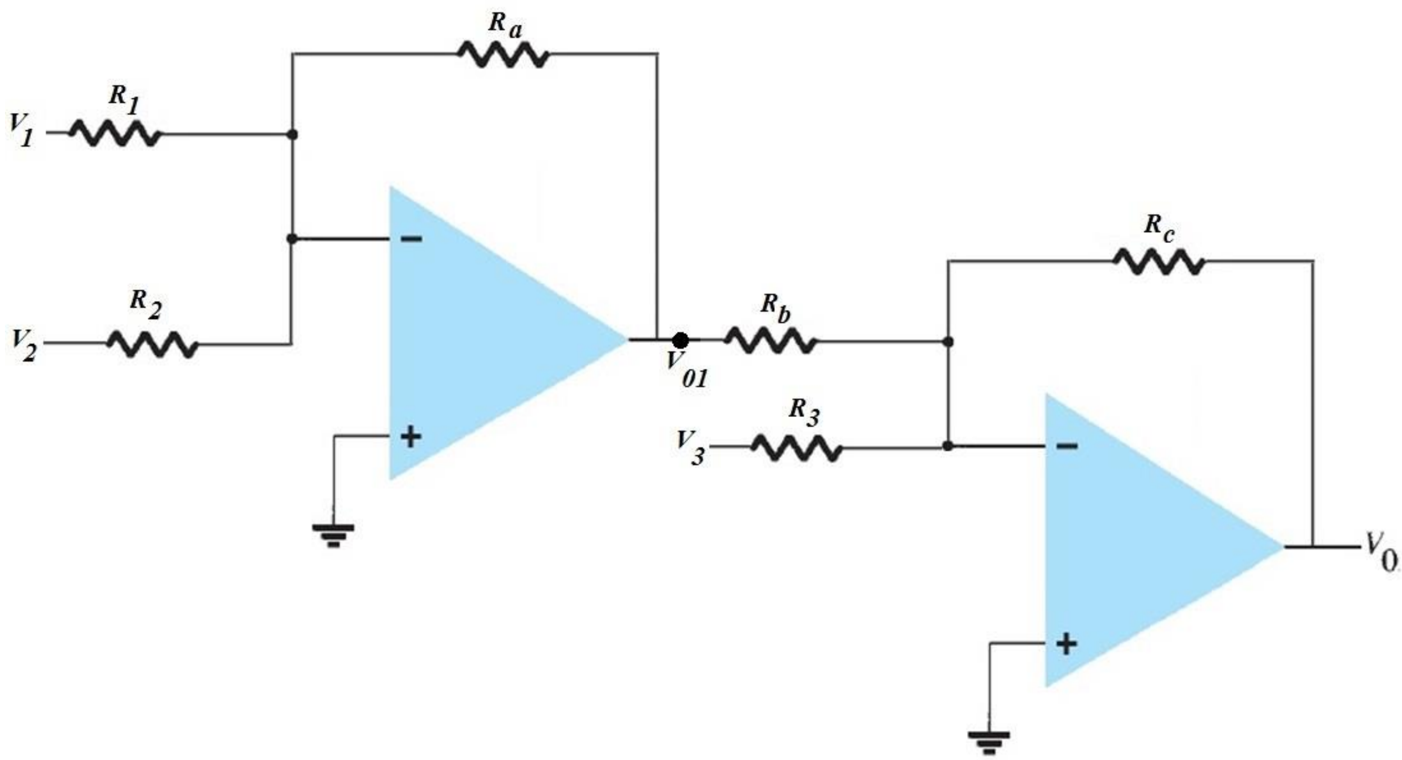
11. Design a summer circuit to get the output as $V_0 = -(V_1 + 5V_2)$. Find the values of R_1, R_2 & R_f so that the maximum output voltage is 10 v. The current in the feedback resistor will not exceed 1 mA.

(Home work)

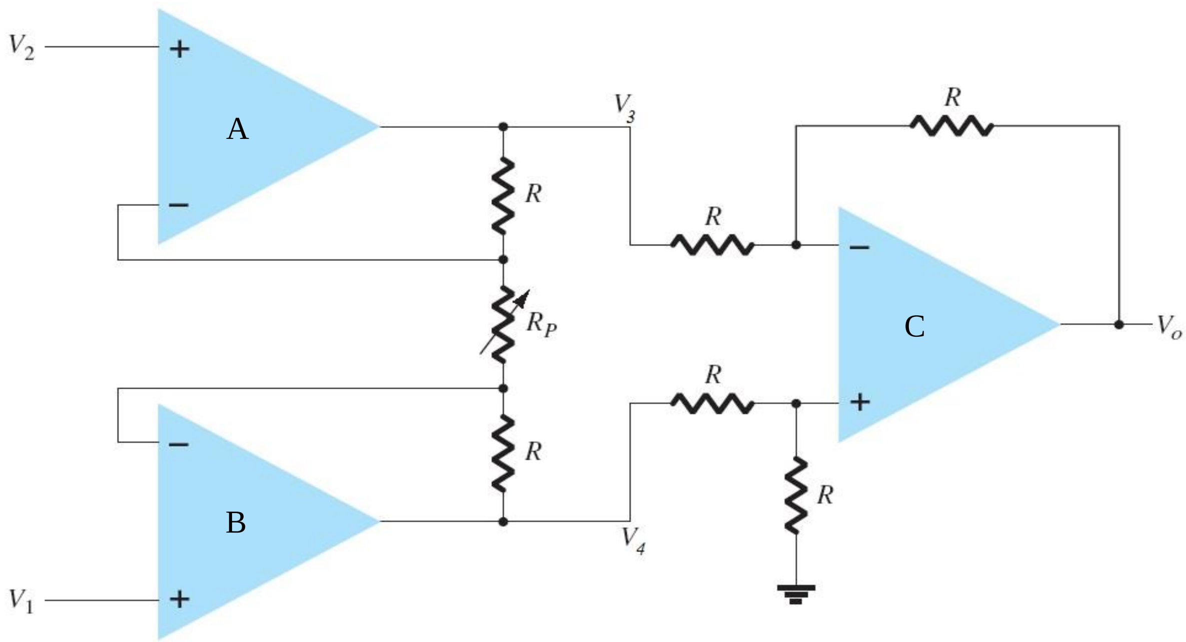
12. Design a circuit using op – amp to get the output as $V_0 = -V_1 + 2V_2 - 3V_3$. Given $R_f = 100\ k\ \Omega$

(Home work)

13. Design the Summer circuit shown below to get the output as $V_0 = 2V_1 + V_2 - 4\ V_3$. Assume Ra = Rb = Rc = 10 k Ω. *(Home work)*



INSTRUMENTATION AMPLIFIER:



Instrumentation amplifier is a combination of three op-amps that are typically grouped into two stages. The first two op-amps comprise the first stage. The second stage is a subtractor circuit. An instrumentation amplifier is beneficial for several reasons:

1. High input impedance, unlike the lower input impedance of a differential amplifier by itself.
2. High CMRR. The source internal resistances of V_1 and V_2 do not affect the total resistance on each input arm.
3. Good for smaller, insignificant input signals.
4. Gain of the first stage can be varied by a variable resistor R_P.

Here, by superposition theorem the output can be written as

$$V_3 = V_{31} + V_{32}$$

where, V_{31} = Output due to the input V_1 when V_2 is at ground potential
V_{32} = Output due to the input V_2 when V_1 is at ground potential

& $$V_4 = V_{41} + V_{42}$$

Where, V_{41} = Output due to the input V_1 when V_2 is at ground potential
V_{42} = Output due to the input V_2 when V_1 is at ground potential

Calculation of V_3:

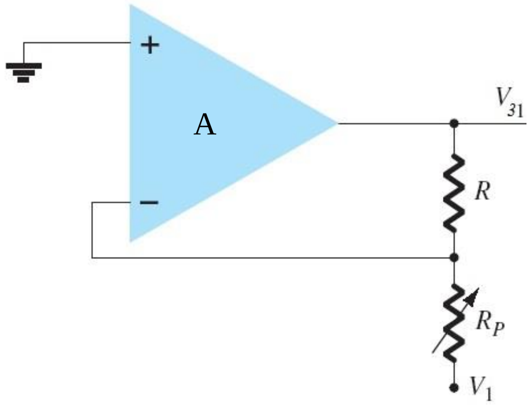
We know that,

$$V_3 = V_{31} + V_{32}$$

Where, V_{31} = Output due to the input V_1 when V_2 is at ground potential
V_{32} = Output due to the input V_2 when V_1 is at ground potential

For V_{31}:

Considering the input V_1 and making the other input V_2 grounded, the circuit for op-amp - A can be drawn as

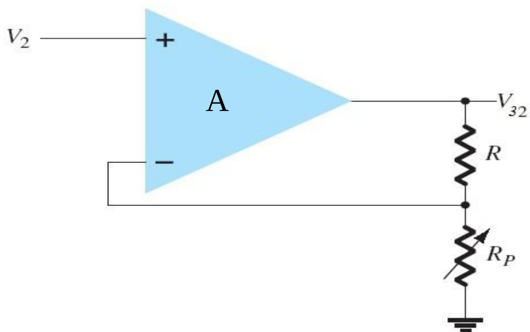


The above circuit is an inverting amplifier & its output can be expressed as,

$$V_{31} = -\frac{R}{R_P}V_1$$

For V_{32}:

Considering the input V_2 and making the other input V_1 grounded, the circuit for op-amp - A can be drawn as



The above circuit is a non- inverting amplifier & its output can be expressed as,

$$V_{32} = \left[1+\frac{R}{R_P}\right]V_2$$

Hence the output V_3 can be expressed as

$$V_3 = V_{31} + V_{32}$$

$$\Rightarrow V_3 = -\frac{R}{R_P}V_1 + \left[1+\frac{R}{R_P}\right]V_2$$

$$\Rightarrow V_3 = -\frac{R}{R_P}V_1 + V_2 + \frac{R}{R_P}V_2$$

Calculation of V_4:

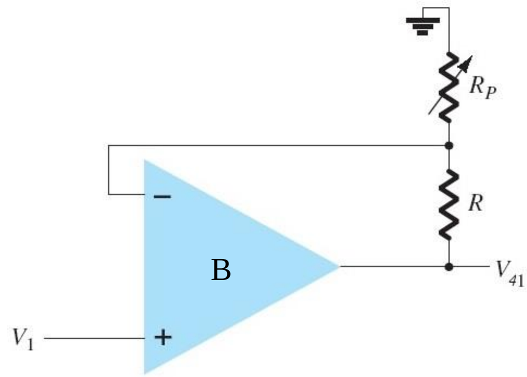
We know that,

$$V_4 = V_{41} + V_{42}$$

where, V_{41} = Output due to the input V_1 when V_2 is at ground potential
V_{42} = Output due to the input V_2 when V_1 is at ground potential

For V_{41}:

Considering the input V_1 and making the other input V_2 grounded, the circuit for op-amp - B can be drawn as

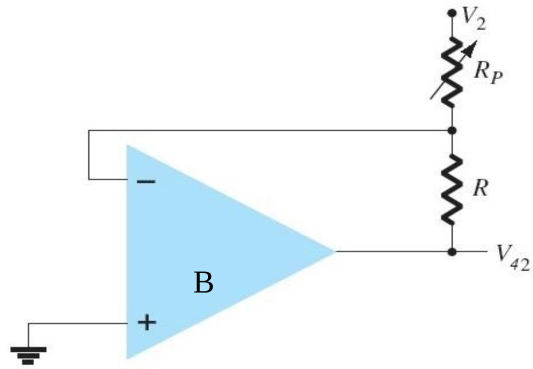


The above circuit is a non- inverting amplifier & its output can be expressed as,

$$V_{41} = \left[1 + \frac{R}{R_P}\right] V_1$$

For V_{42}:

Considering the input V_2 and making the other input V_1 grounded, the circuit for op-amp - B can be drawn as



The above circuit is an inverting amplifier & its output can be expressed as,

$$V_{42} = -\frac{R}{R_P} V_2$$

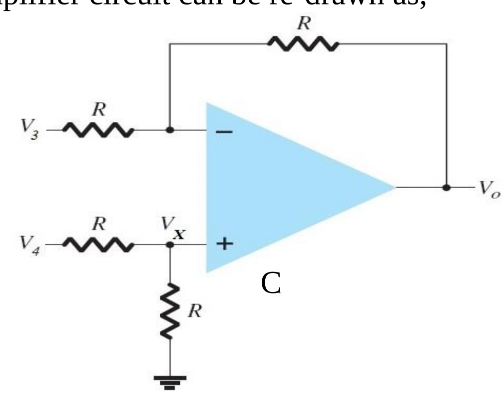
Hence the output V_4 can be expressed as

$$V_4 = V_{41} + V_{42}$$

$$\Rightarrow V_4 = \left[1 + \frac{R}{R_P}\right] V_1 - \frac{R}{R_P} V_2$$

$$\Rightarrow V_4 = V_1 + \frac{R}{R_P} V_1 - \frac{R}{R_P} V_2$$

Now the given instrumentation amplifier circuit can be re-drawn as,

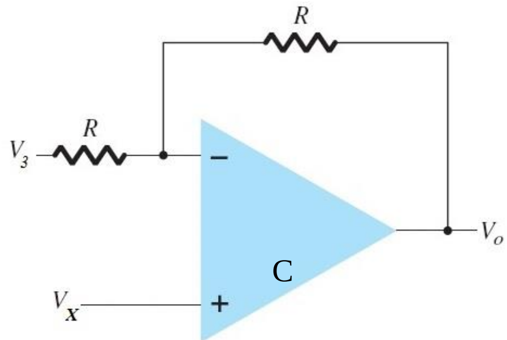


The above circuit is a subtractor circuit and the voltage VX can be calculated by voltage division rule & is given by,

$$V_X = \frac{R}{R+R}V_4$$

$$\Rightarrow V_X = \frac{R}{2R}V_4 = \frac{V_4}{2}$$

Now the above circuit can be re-drawn as,



Here, the output is given by,

$$V_0 = V_{03} + V_{0X}$$

Where, V_{03} = Output due to the input V_3 when V_X is at ground potential
V_{0X}= Output due to the input V_X when V_3 is at ground potential

When we consider the input V_3 by connecting the other input V_X to ground potential, the output is given by

$$V_{03} = -\frac{R}{R}V_3 = -V_3$$

Again when we consider the input V_X by connecting the other input V_3 to ground potential, the output is given by

$$V_{0X} = \left[1 + \frac{R}{R}\right]V_X$$

$$\Rightarrow V_{0X} = \left[1 + \frac{R}{R}\right]\left[\frac{V_4}{2}\right]$$

$$\Rightarrow V_{0X} = V_4$$

Hence the output of the op-amp – C is given by,

$$V_0 = V_{03} + V_{0X}$$

$$\Rightarrow V_0 = -V_3 + V_4$$

$$\Rightarrow V_0 = -\left[-\frac{R}{R_P}V_1 + V_2 + \frac{R}{R_P}V_2\right] + \left[V_1 + \frac{R}{R_P}V_1 - \frac{R}{R_P}V_2\right]$$

$$\Rightarrow V_0 = \frac{R}{R_P}V_1 - V_2 - \frac{R}{R_P}V_2 + V_1 + \frac{R}{R_P}V_1 - \frac{R}{R_P}V_2$$

$$\Rightarrow V_0 = (V_1 - V_2) + 2\frac{R}{R_P}V_1 - 2\frac{R}{R_P}V_2$$

$$\Rightarrow V_0 = (V_1 - V_2) + 2\frac{R}{R_P}(V_1 - V_2)$$

$$\Rightarrow V_0 = (V_1 - V_2)\left[1 + 2\frac{R}{R_P}\right]$$

Now, the above expression can be re-written as,

$$V_0 = K(V_1 - V_2)$$

Where, $K = 1 + 2\frac{R}{R_P}$